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WEIGHTED TURÁN TYPE INEQUALITY FOR RATIONAL FUNCTIONS WITH PRESCRIBED POLES

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Abstract. Firstly, we introduce a new type of weight functions named as N-doubling weights, which is an essential generalization of the well known doubling weights. Secondly, we establish a weighted Turán type inequality with N-doubling weights and a Nikolskii-Turán type inequality for rational functions with prescribed poles. Our results generalize some known Turán type inequality both for polynomials and rational functions.

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