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MATRIX INEQUALITIES INCLUDING GRAND FURUTA INEQUALITY VIA KARCHER MEAN

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Abstract. In our previous paper, we have shown a generalization of Furuta inequality via Karcher mean (Riemannian mean) by using Yamazaki's results which are generalizations of Ando-Hiai inequality and related ones. In this paper, we shall show a generalization of grand Furuta inequality as an extension of our previous result.

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