Journal of
Mathematical
Inequalities
Volume 8, Number 3 (2014), 375–404 doi:10.7153/jmi-08-28

ORIGIN–SYMMETRIC BODIES OF REVOLUTION WITH MINIMAL MAHLER VOLUME IN $\mathbb{R}^3$–A NEW PROOF

YOUJIANG LIN AND GANGSONG LENG

Abstract. In [22], Meyer and Reisner proved the Mahler conjecture for rovelution bodies. In this paper, using a new method, we prove that among *origin-symmetric bodies of revolution* in $\mathbb{R}^3$, cylinders have the minimal Mahler volume. Further, we prove that among *parallel sections homothety bodies* in $\mathbb{R}^3$, 3-cubes have the minimal Mahler volume.

Mathematics subject classification (2010): 52A10, 52A40.
Keywords and phrases: Convex body, body of revolution, polar body, Mahler conjecture, Cylinder.

REFERENCES

[1] S. ARTSTEIN, B. KLARTAG, V. D. MILMAN, *On the Santaló point of a function and a functional Santaló inequality*, Mathematika **54** (2004), 33–48.
[2] K. BALL, *Mahler's conjecture and wavelets*, Discrete Comput. Geom. **13** (1995), 271–277.
[3] J. BOURGAIN, V. D. MILMAN, *New volume ratio properties for convex symmetric bodies in* $\mathbb{R}^n$, Invent. Math. **88** (1987), 319–340.
[4] S. CAMPI, P. GRONCHI, *Volume inequalities for* L_p*-zonotopes*, Mathematika **53** (2006), 71–80.
[5] S. CAMPI AND P. GRONCHI, *On volume product inequalities for convex sets*, Proc. Amer. Math. Soc. **134** (2006), 2393–2402.
[6] S. CAMPI AND P. GRONCHI, *Extremal convex sets for Sylvester-Busemann type functionals*, Appl. Anal. **85** (2006), 129–141.
[7] M. FRADELIZI, Y. GORDON, M. MEYER, S. REISNER, *The case of equality for an inverse Santaló functional inequality*, Adv. Geom., **10** (2010), 621–630.
[8] M. FRADELIZI, M. MEYER, *Some functional forms of Blaschke-Santaló inequality*, Math. Z. **256** (2007), 379–395.
[9] M. FRADELIZI, M. MEYER, *Increasing functions and inverse Santaló inequality for unconditional functions*, Positivity **12** (2008), 407–420.
[10] M. FRADELIZI, M. MEYER, *Some functional inverse Santaló inequalities*, Adv. Math. **218** (2008), 1430–1452.
[11] R. J. GARDNER, *Geometric tomography*, Second edition. Encyclopedia of Mathematics and its Applications, 58. Cambridge University Press, Cambridge, 2006.
[12] Y. GORDON, M. MEYER AND S. REISNER, *Zonoids with minimal volume-product – a new proof*, Proc. Amer. Math. Soc. **104** (1988), 273–276.
[13] J. KIM, S. REISNER, *Local minimality of the volume-product at the simplex*, Mathematika, in press.
[14] G. KUPERBERG, *From the Mahler Conjecture to Gauss Linking Integrals*, Geom. Funct. Anal. **18** (2008), 870–892.
[15] M. A. LOPEZ, S. REISNER, *A Special Case of Mahler's Conjecture*, Discrete Comput. Geom. **20** (1998), 163–177.
[16] E. LUTWAK, D. YANG AND G. ZHANG, *A volume inequality for polar bodies*, J. Differential Geom. **84** (2010) 163–178.
[17] Y. LIN, G. LENG, *Convex bodies with minimal volume product in* $\mathbb{R}^2$ *– a new proof*, Discrete Math. **310** (2010), 3018–3025.
[18] K. MAHLER, *Ein Übertragungsprinzip für konvexe Körper*, Casopis Pest. Mat. Fys. **68** (1939), 93–102.

Paper JMI-08-28

[19] K. MAHLER, *Ein Minimalproblem für konvexe Polygone*, Mathematica (Zutphen) B. **7** (1939), 118–127.

[20] M. MEYER, *Une caractérisation volumique de certains espacés normes de dimension finie*, Israel J. Math. **55** (1986), 317–326.

[21] M. MEYER, *Convex bodies with minimal volume product in* $\mathbb{R}^2$, Monatsh. Math. **112** (1991), 297–301.

[22] M. MEYER AND S. REISNER, *Inequalities involving integrals of Polar-conjugate concave functions*, Monatsh. Math. **125** (1998), 219–227.

[23] M. MEYER AND S. REISNER, *Shadow systems and volumes of polar convex bodies*, Mathematika **53** (2006), 129–148.

[24] F. NAZAROV, F. PETROV, D. RYABOGIN AND A. ZVAVITCH, *A remark on the Mahler conjecture: local minimality of the unit cube*, Duke Math. J. **154** (2010), 419–430.

[25] S. REISNER, *Random polytopes and the volume-product of symmetric convex bodies*, Math. Scand. **57** (1985), 386–392.

[26] S. REISNER, *Zonoids with minimal volume-product*, Math. Z. **192** (1986), 339–346.

[27] S. REISNER, *Minimal volume product in Banach spaces with a 1-unconditional basis*, J. Lond. Math. Soc. **36** (1987), 126–136.

[28] J. SAINT RAYMOND, *Sur le volume des corps convexes sym etriques*, Seminaire d'initiation al Analyse, 1980/1981, Publ. Math. Univ. Pierre et Marie Curie, Paris, 1981, 1–25.

[29] R. SCHNEIDER, *Convex Bodies: The Brunn-Minkowski Theory*, Encyclopedia Math. Appl., vol. 44, Cambridge University Press, Cambridge, 1993.

[30] T. TAO, *Structure and Randomness: pages from year one of a mathematical blog*, Amer. Math. Soc. (2008), 216–219.