SOME HARDY TYPE INEQUALITIES WITH "BROKEN" EXPONENT p

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Abstract. Some new Hardy-type inequalities, where the parameter p is permitted to take different values in different intervals, are proved and discussed. The parameter can even be negative in one interval and greater than one in another. Moreover, a similar result is derived for a multi-dimensional case.

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