

FUNDAMENTAL INEQUALITIES FOR FRACTIONAL HYBRID DIFFERENTIAL EQUATIONS OF DISTRIBUTED ORDER AND APPLICATIONS

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Abstract. In this article, we establish some fundamental strict and non-strict differential inequalities for the fractional hybrid differential equations of distributed order (DOFHDEs). We derive these inequalities with respect to a nonnegative density function in the Riemann-Liouville derivative of order $0 < q < 1$. As an application of these inequalities, we prove the existence results for extremal solution of DOFHDEs and state the comparison principle.

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