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GENERALIZED MINTY PREVARIATIONAL INEQUALITY, INVEX–INCREASE–ALONG–RAYS PROPERTY AND INVEX–STAR–SHAPED OPTIMIZATION PROBLEM

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Abstract. The purpose of this paper is to study some relations between generalized Minty prevariational inequalities, invex-increase-along-rays properties, and invex-star-shaped optimization problems. We introduce the concepts of invex-star-shaped sets and invex-increase-along-rays functions, and establish the relations between invex-increase-along-rays properties and invex-star-shaped optimization problems. Further, under certain conditions, we investigate the relations between invex-increase-along-rays properties and generalized Minty prevariational inequalities. As consequences, we obtain the equivalence of generalized Minty prevariational inequalities and invex-star-shaped optimization problems under suitable conditions. Finally, we prove the equivalence of generalized Minty prevariational inequalities and perturbed generalized Minty prevariational inequalities.

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