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SHARPENING OF THE INEQUALITIES OF SCHUR, EBERLEIN, KRESS AND HUANG, AND NEW LOCATION OF EIGENVALUES

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Abstract. Let $A=(a_{ij})$ be an $n\times n$ complex matrix with eigenvalues $\lambda_1,\lambda_2,\cdots,\lambda_n$. We determine the new upper bounds of $\sum_{j=1}^{n}|\lambda_j|^2$, which will sharpen Schur, Eberlein, Kress and Huang's inequalities. We also exhibit new methods to locate the eigenvalues of a given complex matrix, which are more exact than those existing in previous literature. Numerical examples are provided to show the effectiveness of our results.

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