

## ON EXTENSIONS AND APPLICATIONS OF THE BEESACK INEQUALITY FOR BOUNDING RIEMANN–STIELTJES INTEGRALS

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*Abstract.* A variety of sharp generalisations and extensions of a Beesack inequality are investigated in the current development. A number of applications are examined, including bounding a renewal integral equation which arises in risk/ruin problems. Sharp and tighter bounds than existing results for the Čebyšev functional involving Riemann–Stieltjes integrals are also determined which is of importance in many applications including in providing bounds for perturbed quadrature rules.

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