A NOTE ON GENERALIZED TRIGONOMETRIC AND HYPERBOLIC FUNCTIONS

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Abstract. In this paper, we prove that the function $x \rightarrow \log (x/\sin_p(x))/\log (\sinh_p(x)/x)$ $(p \in [2, \infty))$ is strictly increasing on $(0, \pi_p/2)$, where $\pi_p/2 = \int_0^1 (1-t^p)^{-1/p} dt$, and $\sin_p(x)$ and $\sinh_p(x)$ denote the generalized trigonometric sine and generalized hyperbolic sine functions, respectively. As application, a conjecture due to Klén, Vuorinen and Zhang [J. Math. Anal. Appl. 409 (2014), 521–529] is proved, and the best positive constants $\alpha$ and $\beta$ such that

$$\left( \frac{\sinh_p(x)}{x} \right)^\alpha < \frac{x}{\sin_p(x)} < \left( \frac{\sinh_p(x)}{x} \right)^\beta$$

are determined.

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REFERENCES