A NOTE ON GENERALIZED TRIGONOMETRIC
AND HYPERBOLIC FUNCTIONS

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Abstract. In this paper, we prove that the function $x \to \log(x/\sin_p(x))/\log(\sinh_p(x)/x)$ ($p \in [2,\infty)$) is strictly increasing on $(0, \pi_p/2)$, where $\pi_p/2 = \int_0^1 (1-t^p)^{-1/p} dt$, and $\sin_p(x)$ and $\sinh_p(x)$ denote the generalized trigonometric sine and generalized hyperbolic sine functions, respectively. As application, a conjecture due to Klén, Vuorinen and Zhang [J. Math. Anal. Appl. 409 (2014), 521–529] is proved, and the best positive constants α and β such that

$$\left(\frac{\sinh_p(x)}{x}\right)^{\alpha} < \frac{x}{\sin_p(x)} < \left(\frac{\sinh_p(x)}{x}\right)^{\beta}$$

are determined.

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