FEKETE–SZEGÖ INEQUALITY FOR GENERALIZED
SUBCLASSES OF UNIVALENT FUNCTIONS

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Abstract. Let $\mathscr{P}_{\varphi}(n,b,\lambda)$ denote the class of normalized univalent functions $f(z) = z + a_2 z^2 + ...$, which are defined in the unit disk Δ and satisfying $1 + [(\lambda D^{n+2} f(z) + (1-\lambda) D^{n+1} f(z)) / (\lambda D^{n+1} f(z) + (1-\lambda) D^n f(z)) - 1]/b \prec \varphi(z)$, where $\varphi(z)$ is the function with positive real part, $D^n f$ denotes the sălăgean operator, $n \geqslant 0$, $0 \leqslant \lambda \leqslant 1$, $b \in \mathbb{C}$. In this paper, for the class $\mathscr{P}_{\varphi}(n,b,\lambda)$, the Fekete-Szegö inequalities are completely solved. A more general class $\mathscr{K}(\beta,n,\lambda,g(z))$ related $\mathscr{P}_{\varphi}(n,b,\lambda)$ is also considered with same subject, which extends the earlier corresponding results for the class of strongly close-to-convex functions of order β.

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