

SCHUR M-POWER CONVEXITY OF GENERALIZED HAMY SYMMETRIC FUNCTION

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Abstract. In this paper, we investigate the Schur m -power convexity of the generalized hamy symmetric function

$$F_n^*(\mathbf{x}, r) = \sum_{i_1+i_2+\dots+i_n=r} (x_1^{i_1} x_2^{i_2} \dots x_n^{i_n})^{\frac{1}{r}}$$

for $x \in \mathbb{R}_+^n$ and $r \in \mathbb{N}$ with $1 \leq r \leq n$, which generalizes some known results.

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