

OPERATOR INEQUALITIES AMONG ARITHMETIC MEAN, GEOMETRIC MEAN AND HARMONIC MEAN

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Abstract. We give an upper bound for the weighted geometric mean using the weighted arithmetic mean and the weighted harmonic mean. We also give a lower bound for the weighted geometric mean. These inequalities are proven for two invertible positive operators.

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REFERENCES

- [1] F. KUBO AND T. ANDO, *Means of positive operators*, Math. Ann., Vol. 264 (1980), pp. 205–224.
- [2] T. FURUTA AND M. YANAGIDA, *Generalized means and convexity of inversion for positive operators*, Amer. Math. Monthly, Vol. 105 (1998), pp. 258–259.
- [3] S. FURUICHI, *On refined Young inequalities and reverse inequalities*, J. Math. Ineq., Vol. 5 (2011), pp. 21–31.
- [4] S. FURUICHI, *Refined Young inequalities with Specht's ratio*, J. Egypt. Math. Soc., Vol. 20 (2012), pp. 46–49.
- [5] H. ZUO, G. SHI AND M. FUJII, *Refined Young inequality with Kantorovich constant*, J. Math. Ineq., Vol. 5 (2011), pp. 551–556.
- [6] M. KRNIĆ, N. LOVRIČEVIĆ AND J. PEČARIĆ, *Jensen's operator and applications to mean inequalities for operators in Hilbert space*, Bull. Malays. Math. Sci. Soc., Vol. 35 (2012), pp. 1–14.
- [7] F. KITTANEH, M. KRNIĆ, N. LOVRIČEVIĆ AND J. PEČARIĆ, *Improved arithmetic-geometric and Heinz means inequalities for Hilbert space operators*, Publ. Math. Debrecen, Vol. 80 (2012), pp. 465–478.