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ON THE NORMS OF r–CIRCULANT MATRICES WITH THE HYPER–FIBONACCI AND LUCAS NUMBERS

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Abstract. In this paper, we study norms of circulant matrices $F = \mathrm{Circ}(F_0^{(k)}, F_1^{(k)}, \ldots, F_{n-1}^{(k)})$, $L = \mathrm{Circ}(L_0^{(k)}, L_1^{(k)}, \ldots, L_{n-1}^{(k)})$ and r-circulant matrices $F_r = \mathrm{Circr}(F_0^{(k)}, F_1^{(k)}, \ldots, F_{n-1}^{(k)})$, $L_r = \mathrm{Circr}(L_0^{(k)}, L_1^{(k)}, \ldots, L_{n-1}^{(k)})$, where $F_n^{(k)}$ and $L_n^{(k)}$ denote the hyper-Fibonacci and hyper-Lucas numbers, respectively.

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