

INEQUALITIES INVOLVING GENERALIZED TRIGONOMETRIC AND HYPERBOLIC FUNCTIONS

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Abstract. The Huygens-Wilker type inequalities involving generalized trigonometric functions and generalized hyperbolic functions are established. The first and the second inequalities of Huygens and Wilker, for classes of functions under discussion, are also investigated.

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