

## OPIAL-TYPE INEQUALITIES FOR TWO FUNCTIONS WITH GENERAL KERNELS AND APPLICATIONS

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**Abstract.** In this paper, we establish new Opial-type inequalities for general kernels. We prove the converse of our general result and related extreme case. As applications of our main results we extract the results of [12]. We provide the related applications for Widder's derivative and linear differential operator. At the end, we give the discrete analogue corresponding to our main results.

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