

## SOME INEQUALITIES OF OPERATOR MONOTONE FUNCTIONS

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*Abstract.* Let  $f(t)$  be any non-constant operator monotone function on  $[0, \infty)$  and also let  $A$  and  $B$  be strictly positive operators:

(i) If  $A > B$ , then

$$f(A^\alpha) - f(B^\alpha) \geq f(\|A^\alpha\|) - f\left(\|A^\alpha\| - \frac{1}{\|(A^\alpha - B^\alpha)^{-1}\|}\right) > 0$$

for all  $\alpha \in (0, 1]$ .

(ii) If  $\log A > \log B$ , then there exists  $\beta \in (0, 1]$  such that

$$f(A^\alpha) - f(B^\alpha) \geq f(\|A^\alpha\|) - f\left(\|A^\alpha\| - \frac{1}{\|(A^\alpha - B^\alpha)^{-1}\|}\right) > 0$$

for all  $\alpha \in (0, \beta]$ .

*Mathematics subject classification (2010):* 15A45.

*Keywords and phrases:* Operator monotone function, strictly positive operator, chaotic order, Löwner-Heinz inequality.

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