

SOME INEQUALITIES OF OPERATOR MONOTONE FUNCTIONS

HONGLIANG ZUO AND GUANGCAI DUAN

Abstract. Let $f(t)$ be any non-constant operator monotone function on $[0, \infty)$ and also let A and B be strictly positive operators:

(i) If $A > B$, then

$$f(A^\alpha) - f(B^\alpha) \geq f(\|A^\alpha\|) - f\left(\|A^\alpha\| - \frac{1}{\|(A^\alpha - B^\alpha)^{-1}\|}\right) > 0$$

for all $\alpha \in (0, 1]$.

(ii) If $\log A > \log B$, then there exists $\beta \in (0, 1]$ such that

$$f(A^\alpha) - f(B^\alpha) \geq f(\|A^\alpha\|) - f\left(\|A^\alpha\| - \frac{1}{\|(A^\alpha - B^\alpha)^{-1}\|}\right) > 0$$

for all $\alpha \in (0, \beta]$.

Mathematics subject classification (2010): 15A45.

Keywords and phrases: Operator monotone function, strictly positive operator, chaotic order, Löwner-Heinz inequality.

REFERENCES

- [1] R. BHATIA, *Matrix Analysis*, New York: Springer Verlag, 1997.
- [2] M. FUJII, *Furuta inequality and its related topics*, Ann. Funct. Anal. **1**, 1 (2010), 28–45.
- [3] M. FUJII, J. F. JIANG AND E. KAMEI, *Characterization of chaotic order and its application to Furuta inequality*, Proc. Amer. Math. Soc. **125** (1997), 3655–3658.
- [4] T. FURUTA, *Comprehensive survey on an order preserving operator inequality*, Banach J. Math. Anal. **7**, 1 (2013), 14–40.
- [5] T. FURUTA, *Operator monotone functions, $A > B > 0$ and $\log A > \log B$* , J. Math. Inequal. **7**, 1 (2013), 93–96.
- [6] M. MOSLEHIAN AND H. NAJAFI, *An extension of the Löwner-Heinz inequality*, Linear Algebra Appl. **437** (2012), 2359–2365.
- [7] X. ZHAN, *Matrix Inequalities*, Berlin: Springer Verlag, 2002.