Journal of Mathematical Inequalities
Volume 8, Number 4 (2014), 855–862
doi:10.7153/jmi-08-64

THE SCHUR CONVEXITY FOR THE GENERALIZED MUIRHEAD MEAN

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Abstract. For $x,y>0$, $a,b\in\mathbb{R}$ with $a+b\neq 0$, the generalized Muirhead mean is defined by $M(a,b;x,y)=\left(\frac{x^ay^b+x^by^a}{2}\right)^{\frac{1}{a+b}}$. In this paper, we prove that $M(a,b;x,y)$ is Schur convex with respect to $(x,y)\in(0,\infty)\times(0,\infty)$ if and only if $(a,b)\in\{(a,b)\in\mathbb{R}^2:(a-b)^2\geqslant a+b>0\ \&\ ab\leqslant 0\}$ and Schur concave with respect to $(x,y)\in(0,\infty)\times(0,\infty)$ if and only if $(a,b)\in\{(a,b)\in\mathbb{R}_+^2:(a-b)^2\leqslant a+b\ \&\ (a,b)\neq(0,0)\}\cup\{(a,b)\in\mathbb{R}^2:a+b<0\}$, where $\mathbb{R}_+:=[0,\infty)$.

Mathematics subject classification (2010): Primary 26B25; Secondary 26E60.
Keywords and phrases: Generalized Muirhead mean, Schur convexity, Schur concavity.

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