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PARTIAL SUMS OF GENERALIZED BESSEL FUNCTIONS

HALIT ORHAN AND NIHAT YAGMUR

Abstract. Let $\left(g_{p,b,c}\right)_n(z) = z + \sum_{m=1}^{n} b_m z^{m+1}$ be the sequence of partial sums of generalized and normalized Bessel functions $g_{p,b,c}(z) = z + \sum_{m=1}^{\infty} b_m z^{m+1}$ where $b_m = \frac{(-c/4)^m}{m!(\kappa)_m}$ and $\kappa := p + (b+1)/2 \neq 0, -1, -2, \ldots$. The purpose of the present paper is to determine lower bounds for $\Re\left\{\frac{g_{p,b,c}(z)}{\left(g_{p,b,c}\right)_n(z)}\right\}$, $\Re\left\{\frac{\left(g_{p,b,c}\right)_n(z)}{g_{p,b,c}(z)}\right\}$, $\Re\left\{\frac{g'_{p,b,c}(z)}{\left(g_{p,b,c}\right)'_n(z)}\right\}$ and $\Re\left\{\frac{\left(g_{p,b,c}\right)'_n(z)}{g'_{p,b,c}(z)}\right\}$. Further we give lower bounds for $\Re\left\{\frac{\mathbb{A}[g_{p,b,c}](z)}{\left(\mathbb{A}[g_{p,b,c}]\right)_n(z)}\right\}$ and $\Re\left\{\frac{\left(\mathbb{A}[g_{p,b,c}]\right)_n(z)}{\mathbb{A}[g_{p,b,c}](z)}\right\}$, where $\mathbb{A}[g_{p,b,c}]$ is the Alexander transform of $g_{p,b,c}$.

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