

## $L_p$ -MIXED PROJECTION BODIES AND $L_p$ -MIXED QUERMASSEINTEGRALS

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**Abstract.** In this paper, we research the  $L_p$ -mixed projection bodies by the  $L_p$ -mixed quermass-integrals. First, we give an equivalent conclusion of  $L_p$ -mixed projection bodies. Further, the Shephard type problem for the  $L_p$ -mixed projection bodies are shown.

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