

SOME NORM INEQUALITIES FOR COMMUTATORS WITH SYMBOL FUNCTION IN MORREY SPACES

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Abstract. A version of Dini condition is introduced. Some boundedness of commutators with symbol functions belong to Morrey spaces are discussed. The results can be seen as continuity of our recent work [28].

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