

BEST POSSIBLE INEQUALITIES BETWEEN GENERALIZED LOGARITHMIC MEAN AND WEIGHTED GEOMETRIC MEAN OF GEOMETRIC, SQUARE-ROOT, AND ROOT-SQUARE MEANS

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Abstract. We establish two optimal double inequalities among generalized logarithmic mean $L_p(a, b)$, geometric mean $G(a, b)$, square-root $N(a, b)$, and root-square mean $S(a, b)$.

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