OPTIMAL CONVEX COMBINATIONS BOUNDS OF CENTROIDAL AND HARMONIC MEANS FOR WEIGHTED GEOMETRIC MEAN OF LOGARITHMIC AND IDENTRIC MEANS

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Abstract. In this paper, optimal convex combination bounds of centroidal and harmonic means for weighted geometric mean of logarithmic and identric means are proved. We find the greatest value \( \lambda(\alpha) \) and the least value \( \Delta(\alpha) \) for each \( \alpha \in (0,1) \) such that the double inequality:

\[
\lambda C(a,b) + (1 - \lambda) H(a,b) < L^{\alpha}(a,b) I^{1-\alpha}(a,b) < \Delta C(a,b) + (1 - \Delta) H(a,b)
\]

holds for all \( a,b > 0 \) with \( a \neq b \). Here, \( C(a,b) \), \( H(a,b) \), \( L(a,b) \) and \( I(a,b) \) denote centroidal, harmonic, logarithmic and identric means of two positive numbers \( a \) and \( b \), respectively.


Keywords and phrases: Convex combinations bounds, centroidal mean, harmonic mean, weighted geometric mean, logarithmic mean, identric mean.

REFERENCES