

OPTIMAL CONVEX COMBINATIONS BOUNDS OF CENTROIDAL AND HARMONIC MEANS FOR WEIGHTED GEOMETRIC MEAN OF LOGARITHMIC AND IDENTRIC MEANS

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Abstract. In this paper, optimal convex combination bounds of centroidal and harmonic means for weighted geometric mean of logarithmic and identric means are proved. We find the greatest value $\lambda(\alpha)$ and the least value $\Delta(\alpha)$ for each $\alpha \in (0, 1)$ such that the double inequality:

$$\lambda C(a, b) + (1 - \lambda)H(a, b) < L^\alpha(a, b)I^{1-\alpha}(a, b) < \Delta C(a, b) + (1 - \Delta)H(a, b)$$

holds for all $a, b > 0$ with $a \neq b$. Here, $C(a, b)$, $H(a, b)$, $L(a, b)$ and $I(a, b)$ denote centroidal, harmonic, logarithmic and identric means of two positive numbers a and b , respectively.

Mathematics subject classification (2010): 26D15.

Keywords and phrases: Convex combinations bounds, centroidal mean, harmonic mean, weighted geometric mean, logarithmic mean, identric mean.

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