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UPPER AND LOWER BOUNDS FOR THE p–ANGULAR DISTANCE IN NORMED SPACES WITH APPLICATIONS

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Abstract. For nonzero vectors x and y in the normed linear space $(X, \|\cdot\|)$ we can define the p*-angular distance* by

$$\alpha_p[x,y] := \left\| \|x\|^{p-1} x - \|y\|^{p-1} y \right\|.$$

In this paper we show among others that

$$\begin{aligned} &\frac{1}{2} \left| \left| \|x\|^{p-1} - \|y\|^{p-1} \right| \|x+y\| - \left(\|x\|^{p-1} + \|y\|^{p-1} \right) \|x-y\| \right| \\ &\leqslant \alpha_p[x,y] \\ &\leqslant \frac{1}{2} \left[\left| \|x\|^{p-1} - \|y\|^{p-1} \right| \|x+y\| + \left(\|x\|^{p-1} + \|y\|^{p-1} \right) \|x-y\| \right] \end{aligned}$$

for any $p \in \mathbb{R}$ and for any nonzero $x, y \in X$.

Some reverses of the triangle and the continuity of the norm inequalities are given as well.

Applications for functions f defined by power series in estimating the more general "distance" $\|f(\|x\|)x - f(\|y\|)y\|$ for certain $x, y \in X$ are also provided.

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