

ADDITIVE ρ -FUNCTIONAL INEQUALITIES AND EQUATIONS

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Abstract. In this paper, we investigate the additive ρ -functional inequalities

$$\left\| f\left(\sum_{j=1}^k x_j\right) - \sum_{j=1}^k f(x_j) \right\| \leq \left\| \rho\left(kf\left(\frac{\sum_{j=1}^k x_j}{k}\right) - \sum_{j=1}^k f(x_j)\right) \right\| \quad (0.1)$$

and

$$\left\| kf\left(\frac{\sum_{j=1}^k x_j}{k}\right) - \sum_{j=1}^k f(x_j) \right\| \leq \left\| \rho\left(f\left(\sum_{j=1}^k x_j\right) - \sum_{j=1}^k f(x_j)\right) \right\|, \quad (0.2)$$

where ρ is a fixed complex number with $|\rho| < 1$.

Furthermore, we prove the Hyers-Ulam stability of the additive ρ -functional inequalities (0.1) and (0.2) in complex Banach spaces and prove the Hyers-Ulam stability of additive ρ -functional equations associated with the additive ρ -functional inequalities (0.1) and (0.2) in complex Banach spaces.

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