

A COMPLETELY MONOTONIC FUNCTION RELATING TO THE q -TRIGAMMA FUNCTION

JIAO-LIAN ZHAO

Abstract. In the paper, a function relating to the q -trigamma function is proved to be completely monotonic. As by-products, two functions relating to the logarithmic function are also proved to be completely monotonic.

Mathematics subject classification (2010): Primary 33D05; Secondary 26A48.

Keywords and phrases: Completely monotonic function, q -trigamma function, logarithmic function.

REFERENCES

- [1] G. D. ANDERSON, R. W. BARNARD, K. C. RICHARDS, M. K. VAMANAMURTHY, AND M. VUORINEN, *Inequalities for zero-balanced hypergeometric functions*, Trans. Amer. Math. Soc. **347** (1995), no. 5, 1713–1723; Available online at <http://dx.doi.org/10.1090/S0002-9947-1995-1264800-3>.
- [2] G. E. ANDREWS, R. A. ASKEY, AND R. ROY, *Special Functions*, Cambridge University Press, Cambridge, 1999.
- [3] C.-P. CHEN AND F. QI, *Logarithmically completely monotonic functions relating to the gamma function*, J. Math. Anal. Appl. **321** (2006), no. 1, 405–411; Available online at <http://dx.doi.org/10.1016/j.jmaa.2005.08.056>.
- [4] B.-N. GUO AND F. QI, *Properties and applications of a function involving exponential functions*, Commun. Pure Appl. Anal. **8** (2009), no. 4, 1231–1249; Available online at <http://dx.doi.org/10.3934/cpaa.2009.8.1231>.
- [5] M. E. H. ISMAIL, L. LORCH, AND M. E. MULDOON, *Completely monotonic functions associated with the gamma function and its q -analogues*, J. Math. Anal. Appl. **116** (1986), 1–9; Available online at [http://dx.doi.org/10.1016/0022-247X\(86\)90042-9](http://dx.doi.org/10.1016/0022-247X(86)90042-9).
- [6] M. E. H. ISMAIL AND M. E. MULDOON, *Inequalities and monotonicity properties for gamma and q -gamma functions*, in: R.V.M. Zahar (Ed.), *Approximation and Computation: A Festschrift in Honour of Walter Gautschi*, ISNM, Vol. **119**, BirkhRausser, Basel, 1994, 309–323.
- [7] M. E. H. ISMAIL AND M. E. MULDOON, *Inequalities and monotonicity properties for gamma and q -gamma functions*, available online at <http://arxiv.org/abs/1301.1749>.
- [8] D. S. MITRINOVIĆ, J. E. PEČARIĆ AND A. M. FINK, *Classical and New Inequalities in Analysis*, Kluwer Academic Publishers, Dordrecht-Boston-London, 1993.
- [9] M. E. MULDOON, *Some monotonicity properties and characterizations of the gamma function*, Aequationes Math. **18** (1978), 54–63; Available online at <http://dx.doi.org/10.1007/BF01844067>.
- [10] F. QI, *A completely monotonic function involving the divided difference of the psi function and an equivalent inequality involving sums*, ANZIAM J. **48** (2007), no. 4, 523–532; Available online at <http://dx.doi.org/10.1017/S1446181100003199>.
- [11] F. QI, *A completely monotonic function related to the q -trigamma function*, Politehn. Univ. Bucharest Sci. Bull. Ser. A Appl. Math. Phys. **75** (2013), in press.
- [12] F. QI, *Bounds for the ratio of two gamma functions*, J. Inequal. Appl. **2010** (2010), Article ID 493058, 84 pages; Available online at <http://dx.doi.org/10.1155/2010/493058>.
- [13] F. QI, *Three classes of logarithmically completely monotonic functions involving gamma and psi functions*, Integral Transforms Spec. Funct. **18** (2007), no. 7, 503–509; Available online at <http://dx.doi.org/10.1080/10652460701358976>.

- [14] F. QI AND B.-N. GUO, *Completely monotonic functions involving divided differences of the di- and tri-gamma functions and some applications*, *Commun. Pure Appl. Anal.* **8** (2009), no. 6, 1975–1989; Available online at <http://dx.doi.org/10.3934/cpaa.2009.8.1975>.
- [15] F. QI AND B.-N. GUO, *Necessary and sufficient conditions for functions involving the tri- and tetra-gamma functions to be completely monotonic*, *Adv. Appl. Math.* **44** (2010), no. 1, 71–83; Available online at <http://dx.doi.org/10.1016/j.aam.2009.03.003>.
- [16] F. QI AND B.-N. GUO, *Some completely monotonic functions involving the q-tri- and -tetra-gamma functions and applications*, Available online at <http://arxiv.org/abs/1301.0155>.
- [17] D. V. WIDDER, *The Laplace Transform*, Princeton University Press, Princeton, 1946.