OPTIMAL INEQUALITIES FOR THE CONVEX COMBINATION OF ERROR FUNCTION

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Abstract. For $\lambda \in (0,1)$ and $x, y > 0$ we obtain the best possible constants $p$ and $r$, such that
$$\text{erf}(M_p(x, y; \lambda)) \leq \lambda \text{erf}(x) + (1 - \lambda) \text{erf}(y) \leq \text{erf}(M_r(x, y; \lambda))$$
where $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ and $M_p(x, y; \lambda) = (\lambda x^p + (1 - \lambda) y^p)^{1/p} (p \neq 0)$, $M_0(x, y; \lambda) = x^\lambda y^{1-\lambda}$ are error function and weighted power mean, respectively. Furthermore, using these results, we generalized and complement an inequality due to Alzer.


Keywords and phrases: Error function, power mean, functional inequalities.

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