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SOME GENERALIZATIONS OF OPERATOR INEQUALITIES

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Abstract. In this paper, we generalize some operator inequalities as follows: Let $A,\ A_i\ \ (i=1,\dots,n)$ be positive operators on a Hilbert space with $0<m\leqslant A,\ A_i\leqslant M\ \ (i=1,\dots,n)$. Then for $1\leqslant p<\infty$ and every positive unital linear map Φ,

$$\Phi^p(A^{-1})\Phi^p(A)+\Phi^p(A)\Phi^p(A^{-1})\leqslant\frac{(M+m)^{2p}}{2M^pm^p},$$

and

$$\left(\frac{A_1+\cdots+A_n}{n}\right)^{2p}\leqslant\left(\frac{(M+m)^{2p}}{4M^pm^p}\right)^2G^{2p}(A_1,\dots,A_n),$$

where $G(A_1,\dots,A_n)$ is Ando-Li-Mathias geometric mean [1].

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