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NEUMAN–SÁNDOR MEAN, ASYMPTOTIC EXPANSIONS AND RELATED INEQUALITIES

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Abstract. The subject of this paper is a systematic study of inequalities of the form

$$(1-\mu)M_1+\mu M_3 \leqslant M_2 \leqslant (1-\nu)M_1+\nu M_3$$

which cover Neuman-Sándor mean and some classical means. Furthermore, following inequalities with optimal parameters were proved:

$$\mu\frac{1}{H(s,t)}+(1-\mu)\frac{1}{NS(s,t)} \leqslant \frac{1}{A(s,t)} \leqslant \nu\frac{1}{H(s,t)}+(1-\nu)\frac{1}{NS(s,t)}$$

and

$$\mu\frac{1}{H(s,t)}+(1-\mu)\frac{1}{N(s,t)} \leqslant \frac{1}{NS(s,t)} \leqslant \nu\frac{1}{H(s,t)}+(1-\nu)\frac{1}{N(s,t)}.$$

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Paper JMI-09-102

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