

HOW DO SINGULARITIES OF FUNCTIONS AFFECT THE CONVERGENCE OF q -BERNSTEIN POLYNOMIALS?

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Abstract. In this article, the approximation of functions with a singularity at $\alpha \in (0, 1)$ by the q -Bernstein polynomials for $q > 1$ has been studied. Unlike the situation when $\alpha \in (0, 1) \setminus \{q^{-j}\}_{j \in \mathbb{N}}$, in the case when $\alpha = q^{-m}$, $m \in \mathbb{N}$, the type of singularity has a decisive effect on the set where a function can be approximated. In the latter event, depending on the types of singularities, three classes of functions have been examined, and it has been found that the possibility of approximation varies considerably for these classes.

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