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CONTRACTIONS AND THE SPECTRAL CONTINUITY FOR k–QUASI–PARANORMAL OPERATORS

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Abstract. For a positive integer k, an operator $T \in B(\mathscr{H})$ is called k-quasi-paranormal if $\|T^{k+1}x\|^2 \leqslant \|T^{k+2}x\|\|T^k x\|$ for all $x \in \mathscr{H}$, which is a common generalization of paranormal and quasi-paranormal. In this paper, firstly we prove that if T is a contraction of k-quasi-paranormal operators, then either T has a nontrivial invariant subspace or T is a proper contraction and the nonnegative operator $D_\lambda = T^{*k}(|T^2|^2 - 2\lambda|T|^2 + \lambda^2 I)T^k$ for $0 < \lambda \leqslant 1$ is a strongly stable contraction; secondly we prove that k-quasi-paranormal operators are not supercyclic; at last we prove that the spectrum is continuous on the class of all k-quasi-paranormal operators.

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