

ON SCHUR m -POWER CONVEXITY FOR RATIOS OF SOME MEANS

HONG-PING YIN, HUAN-NAN SHI AND FENG QI

Abstract. In the paper, the authors discuss the Schur m -power convexity on $(0, \infty) \times (0, \infty)$ for ratios of some famous means, such as the arithmetic, geometric, harmonic, root-square means, and the like, and obtain some inequalities related to ratios of means.

Mathematics subject classification (2010): Primary 26B25; Secondary 26E60, 26D20.

Keywords and phrases: Schur m -power convexity, quotient, mean, arithmetic mean, geometric mean, harmonic mean.

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