

## ASYMPTOTIC INEQUALITIES AND COMPARISON OF CLASSICAL MEANS

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*Abstract.* Inequalities between linear combinations of means are the subject of interest for decades. In this paper we propose a new approach to this subject, using the concept of asymptotical expansion of means. This enables us to find necessary conditions and the optimal values for coefficients in order that the inequality between linear combination of three or four means will be valid. We restrict ourselves to the study of the most common classical mean, leaving detailed study of parametric means to the future works.

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