Journal of Mathematical Inequalities
Volume 9, Number 1 (2015), 203–217
doi:10.7153/jmi-09-19

SHARP INEQUALITIES FOR TRIGONOMETRIC AND HYPERBOLIC FUNCTIONS

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Abstract. We establish several sharp inequalities for trigonometric and hyperbolic functions. Our results sharpen some known inequalities.
Mathematics subject classification (2010): 26D05, 26D07.
Keywords and phrases: Inequalities, trigonometric functions, hyperbolic functions.

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