NEW GENERALIZED 2D NONLINEAR INEQUALITIES AND APPLICATIONS IN FRACTIONAL DIFFERENTIAL–INTEGRAL EQUATIONS

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Abstract. In this paper, we *study* some new generalized 2D nonlinear Gronwall-Bellman type inequalities, which provide explicit bounds for unknown functions concerned, and are useful in the analysis of qualitative and quantitative properties for solutions to fractional differential and differential-integral equations. The presented inequalities are of new forms compared with the existing results so far in the literature. For illustrating the validity of the results presented, we present one application for them, in which the boundedness, uniqueness, and continuous dependence on the initial value and parameter for the solution to a certain fractional differential-integral equation are investigated.

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