Journal of
Mathematical
Inequalities
Volume 9, Number 1 (2015), 257–276 doi:10.7153/jmi-09-23

PARABOLIC FRACTIONAL MAXIMAL AND INTEGRAL OPERATORS WITH ROUGH KERNELS IN PARABOLIC GENERALIZED MORREY SPACES

VAGIF S. GULIYEV AND AYDIN S. BALAKISHIYEV

Abstract. Let P be a real $n\times n$ matrix, whose all the eigenvalues have positive real part, $A_t = t^P$, $t>0$, $\gamma = \mathrm{tr}P$ is the homogeneous dimension on $\mathbb{R}^n$ and Ω is an A_t-homogeneous of degree zero function, integrable to a power $s>1$ on the unit sphere generated by the corresponding parabolic metric. We study the parabolic fractional maximal and integral operators $M^P_{\Omega,\alpha}$ and $I^P_{\Omega,\alpha}$, $0<\alpha<\gamma$ with rough kernels in the parabolic generalized Morrey space $\mathscr{M}_{p,\varphi,P}(\mathbb{R}^n)$. We find conditions on the pair (φ_1,φ_2) for the boundedness of the operators $M^P_{\Omega,\alpha}$ and $I^P_{\Omega,\alpha}$ from the space $\mathscr{M}_{p,\varphi_1,P}(\mathbb{R}^n)$ to another one $\mathscr{M}_{q,\varphi_2,P}(\mathbb{R}^n)$, $1<p<q<\infty$, $1/p-1/q=\alpha/\gamma$, and from the space $\mathscr{M}_{1,\varphi_1,P}(\mathbb{R}^n)$ to the weak space $W\mathscr{M}_{q,\varphi_2,P}(\mathbb{R}^n)$, $1\leqslant q<\infty$, $1-1/q=\alpha/\gamma$. We also find conditions on φ for the validity of the Adams type theorems $M^P_{\Omega,\alpha}$, $I^P_{\Omega,\alpha}$: $\mathscr{M}_{p,\varphi^{\frac{1}{p}},P}(\mathbb{R}^n)\to\mathscr{M}_{q,\varphi^{\frac{1}{q}},P}(\mathbb{R}^n), 1<p<q<\infty$.

Mathematics subject classification (2010): 42B20, 42B25, 42B35.
Keywords and phrases: Parabolic fractional maximal function, parabolic fractional integral, parabolic generalized Morrey space.

REFERENCES

[1] D. R. ADAMS, *A note on Riesz potentials*, Duke Math. **42** (1975), 765–778.
[2] A. AKBULUT, V. S. GULIYEV, R. MUSTAFAYEV, *On the boundedness of the maximal operator and singular integral operators in generalized Morrey spaces*, Math. Bohem. **137**, 1 (2012), 27–43.
[3] O. V. BESOV, V. P. IL'IN, P. I. LIZORKIN, *The L_p-estimates of a certain class of non-isotropically singular integrals*, (Russian) Dokl. Akad. Nauk SSSR **169** (1966), 1250–1253.
[4] V. I. BURENKOV, H. V. GULIYEV, AND V. S. GULIYEV, *Necessary and sufficient conditions for boundedness of the fractional maximal operators in the local Morrey-type spaces*, J. Comput. Appl. Math. **208**, 1 (2007), 280–301.
[5] V. BURENKOV, A. GOGATISHVILI, V. S. GULIYEV, R. MUSTAFAYEV, *Boundedness of the fractional maximal operator in local Morrey-type spaces*, Complex Var. Elliptic Equ. **55**, 8–10 (2010), 739–758.
[6] V. BURENKOV, A. GOGATISHVILI, V. S. GULIYEV, R. MUSTAFAYEV, *Boundedness of the Riesz potential in local Morrey-type spaces*, Potential Anal. **35**, 1 (2011), 67–87.
[7] A. P. CALDEREON, A. TORCHINSKY, *Parabolic maximal function associated with a distribution, I.*, Advances in Math. **16** (1975), 1–64.
[8] R. R. COIFMAN, G. WEISS, *Analyse harmonique non-commutative sur certains espaces homogènes*, (French) Étude de certaines intégrales singulières, Lecture Notes in Mathematics, Vol. 242. Springer-Verlag, Berlin-New York, 1971.
[9] E. B. FABES, N. RIVÈRE, *Singular integrals with mixed homogeneity*, Studia Math. **27** (1966), 19–38.
[10] G. B. FOLLAND, E. M. STEIN, *Hardy Spaces on Homogeneous Groups*, Math. Notes, 28, Princeton Univ. Press, Princeton, 1982.
[11] Y. DING, D. YANG, Z. ZHOU, *Boundedness of sublinear operators and commutators on $L^{p,\omega}(\mathbb{R}^n)$*, Yokohama Math. J. **46** (1998), 15–27.

Paper JMI-09-23

[12] M. GIAQUINTA, *Multiple integrals in the calculus of variations and nonlinear elliptic systems*, Princeton Univ. Press, Princeton, NJ, 1983.

[13] V. S. GULIYEV, *Integral operators on function spaces on the homogeneous groups and on domains in* $\mathbb{R}^n$, Doctor's degree dissertation, Mat. Inst. Steklov, Moscow, 1994, 329 pp. (in Russian).

[14] V. S. GULIYEV, *Function spaces, integral operators and two weighted inequalities on homogeneous groups. Some applications*, Casioglu, Baku, 1999, 332 pp. (in Russian).

[15] V. S. GULIYEV, *Boundedness of the maximal, potential and singular operators in the generalized Morrey spaces*, J. Inequal. Appl. **2009**, Art. ID 503948, 1–20.

[16] V. S. GULIYEV, R. MUSTAFAYEV, *Boundedness of the anisotropic maximal and anisotropic singular integral operators in generalized Morrey spaces*, Acta Mathematica Sinica (English series) **27**, 12 (2011), 2361–2370.

[17] V. S. GULIYEV, S. S. ALIYEV, T. KARAMAN, AND P. S. SHUKUROV, *Boundedness of sublinear operators and commutators on generalized Morrey Space*, Integral Equat. Oper. Theor. **71**, 3 (2011), 327–355.

[18] V. S. GULIYEV, *Generalized local Morrey spaces and fractional integral operators with rough kernel*, J. Math. Sci. (N.Y.) **193**, 2 (2013), 211–227.

[19] V. S. GULIYEV, L. SOFTOVA, *Global regularity in generalized Morrey spaces of solutions to nondivergence elliptic equations with VMO coefficients*, Potential Anal. **38**, 4 (2013), 843–862.

[20] A. MAUGERI, D. K. PALAGACHEV, L. G. SOFTOVA, *Elliptic and Parabolic Equations with Discontinuous Coefficients*, Wiley-VCH, Berlin 2000.

[21] T. MIZUHARA, *Boundedness of some classical operators on generalized Morrey spaces*, In: Harmonic Analysis, Proc. Conf., Sendai/Jap. 1990, ICM-90 Satell. Conf. Proc., pp. 183–189 (1991).

[22] C. B. MORREY, *On the solutions of quasi-linear elliptic partial differential equations*, Trans. Amer. Math. Soc. **43** (1938), 126–166.

[23] B. MUKENHOUPT AND R. L. WHEEDEN, *Weighted norm inequalities for singular and fractional integral*, Trans. Amer. Math. Sco. **161** (1971), 249–258.

[24] E. NAKAI, *Hardy-Littlewood maximal operator, singular integral operators and the Riesz potentials on generalized Morrey spaces*, Math. Nachr. **166** (1994), 95–103.

[25] E. NAKAI, *The Campanato, Morrey and Hölder spaces on spaces of homogeneous type*, Studia Math. **176**, 1 (2006), 1–19.

[26] L.-E. PERSSON, N. SAMKO, *Weighted Hardy and potential operators in the generalized Morrey spaces*, J. Math. Anal. Appl. **377**, 2 (2011), 792–806.

[27] E. M. STEIN, *Harmonic Analysis: Real Variable Methods, Orthogonality and Oscillatory Integrals*, Princeton Univ. Press, Princeton NJ, 1993.

[28] E. M. STEIN AND S. WAINGER, *Problems in harmonic analysis related to curvature*, Bull. Amer. Math. Soc. **84** (1978), 1239–1295.

[29] NA WEI, PENGCHENG NIU, SUFANG TANG, MAOCHUN ZHU, *Estimates in generalized Morrey spaces for nondivergence degenerate elliptic operators with discontinuous coefficients*, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Math. RACSAM **106**, 1 (2012), 1–33.

[30] LIN ZHANG, YIN SHENG JIANG, JIANG ZHOU, *Parabolic equations with VMO coefficients in generalized Morrey spaces*, Acta Math. Sin. (Engl. Ser.) **26**, 1 (2010), 117–130.