

## MULTIPLE OPIAL-TYPE INEQUALITIES FOR GENERAL KERNELS WITH APPLICATIONS

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*Abstract.* The main purpose of this paper is to give the general multiple Opial-type inequalities for general kernels. We consider the monotocity and boundedness of the weight functions to prove new inequalities. As applications of our general results we establish new inequalities for Widder's derivatives and linear differential operator. Results from [11] are obtain by applying the Canavati fractional derivatives to our main results.

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