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ADDITIVE ρ–FUNCTIONAL INEQUALITIES IN NON–ARCHIMEDEAN NORMED SPACES

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Abstract. In this paper, we solve the additive ρ-functional inequalities

$$\|f(x+y)-f(x)-f(y)\| \leqslant \left\|\rho\left(2f\left(\frac{x+y}{2}\right)-f(x)-f(y)\right)\right\| \tag{0.1}$$

and

$$\left\|2f\left(\frac{x+y}{2}\right)-f(x)-f(y)\right\| \leqslant \|\rho\,(f(x+y)-f(x)-f(y))\|, \tag{0.2}$$

where ρ is a fixed non-Archimedean number with $|\rho|<1$.

Furthermore, we prove the Hyers-Ulam stability of the additive ρ-functional inequalities (0.1) and (0.2) in non-Archimedean Banach spaces and prove the Hyers-Ulam stability of additive ρ-functional equations associated with the additive ρ-functional inequalities (0.1) and (0.2) in non-Archimedean Banach spaces.

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