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A NOTE ON JORDAN–VON NEUMANN CONSTANT FOR $Z_{p,q}$ SPACE

CHANGSEN YANG

Abstract. Let $\lambda > 0$, $Z_{p,q}$ denote $\mathbb{R}^2$ endowed with the norm

$$|x|_{p,q} = (\|x\|_p^2 + \lambda\|x\|_q^2)^{\frac{1}{2}}.$$

Recently, von Neumann-Jordan constant $C_{NJ}(Z_{p,q})$ have been investigated under the two cases of a space $2 \leqslant p \leqslant q \leqslant \infty$ and $1 \leqslant p \leqslant q \leqslant 2$. For the case of $1 \leqslant p < 2 < q \leqslant \infty$, we only have shown an inequality on the constant. In this note, the exact value of the Jordan-von Neumann constant about this case is investigated.

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REFERENCES

[1] J. ALOSNSO, P. MATRÍN AND P. L. PAPINI, *Wheeling around von Neumann-Jordan constant in Banach spaces*, Studia Math. **188** (2008), 135–152.
[2] J. A. CLARKSON, *The von Neumann-Jordan constant for the Lebesgue space*, Ann. of Math. **38** (1937), 114–115.
[3] J. GAO, *A Pythagorean approach in Banach spaces*, J. Inequal. Appl. (2006), Art. ID 94982, 41–56.
[4] J. GAO AND K. S. LAU, *On the geometry of spheres in normaled linear spaces*, J. Aust. Math. Soc. **48** (1990), 101–112.
[5] J. GAO AND S. SAEJUNG, *Some geometric measures of spheres in Banach spaces*, Appl. Math. Comput. **214** (2009), 102–107.
[6] M. KATO, L. MALIGRANDA AND Y. TAKAHASHI, *On James and Jordan-von Neumann constants and the normal structure coefficient of Banach spaces*, Studia Math. **144** (2001), 275–292.
[7] F. F. BONSALL AND J. DUNCAN, *Numerical Ranges II*, Cambridge University Press, 1973.
[8] H. MIZUGUCHI AND K.-S. SAITO, *Some geometric constants of absolute normalized norms on* $\mathbb{R}^2$, Ann. Funct. Anal. **2** (2011), 22–33.
[9] K.-S. SATIO, M. KATO AND Y. TAKAHASHI, *Von Neumann-Jordan constant of absolute normalized norms on* $\mathbb{C}^2$, J. Math. Anal. Appl. **244**, 2 (2000), 515–532.
[10] C. YANG AND F. WANG, *On a new geometric constant related to the von Neumann-Jordan constant*, J. Math. Anal. Appl. **324** (2006), 555–565.
[11] C. YANG AND H. LI, *An inequality on Jordan-von Neumann constant and James constant on* $Z_{p,q}$ *space*, J. Math. Inq. **7**, 1 (2013), 97–102.
[12] C. YANG AND H. LI, *An inequality between Jordan-von Neumann constant and James constant*, Appl. Math. Lett. **23**, 3 (2010), 277–281.

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