SHARP INEQUALITIES INVOLVING
NEUMAN MEANS OF THE SECOND KIND

WEI-MAO QIAN, ZHI-HUA SHAO AND YU-MING CHU

Abstract. In the article, we present several sharp inequalities involving the Neuman means of the second kind, and the logarithmic, Seiffert, arithmetic, Neuman-Sándor, quadratic and contra-harmonic means.

Mathematics subject classification (2010): 26E60.

Keywords and phrases: Neuman means of the second kind, logarithmic mean, Seiffert mean, quadratic mean, Neuman-Sándor mean, contraharmonic mean.

REFERENCES

[1] B. C. CARLSON, *Special Functions of Applied Mathematics*, Academic Press, New York, 1977.
[2] B. C. CARLSON AND J. L. GUSTAFSON, *Asymptotic approximations for symmetric elliptic integrals*, SIAM J. Math. Anal., 1994, **25** (2): 288–303.
[3] E. NEUMAN AND J. SÁNDOR, *On the Schwab-Borchardt mean*, Math. Pannon., 2003, **14** (2): 253–266.
[4] E. NEUMAN AND J. SÁNDOR, *On the Schwab-Borchardt mean II*, Math. Pannon., 2006, **17** (1): 49–59.
[5] E. NEUMAN, *Inequalities for the Schwab-Boorchardt mean and their applications*, J. Math. Inequal., 2011, **5** (4): 601–609.
[6] B. C. CARLSON, *Algorithms involving arithmetic and geometric means*, Amer. Math. Monthly, 1971, **78**: 496–505.
[7] J. L. BRENNER AND B. C. CARLSON, *Homogeneous mean values: weights and asymptotics*, J. Math. Anal. Appl., 1987, **123** (1): 265–280.
[8] H. LIU AND X.-J. MENG, *The optimal convex combination bounds for Seiffert's mean*, J. Inequal. Appl., 2011, Article ID 686834, 9 pages.
[9] W.-D. JIANG AND F. QI, *Some sharp inequalities involving Seiffert and other means and their concise proofs*, Math. Inequal. Appl., 2012, **15** (4): 1007–1017.
[10] W.-D. JIANG, *Some sharp inequalities involving reciprocals of the Seiffert and other means*, J. Math. Inequal., 2012, **6** (4): 593–599.
[11] E. NEUMAN, *A note on a certain bivariate mean*, J. Math. Inequal., 2012, **6** (4): 637–643.
[12] E. NEUMAN, *On one-parameter family of bivariate means*, Aequations Math., 2012, **83** (1–2): 191–197.
[13] E. NEUMAN, *A one-parameter family of bivariate means*, J. Math. Inequal., 2013, **7** (3): 399–412.
[14] E. NEUMAN, *Inequalities involving certain bivariate means II*, J. Inequal. Spec. Funct., 2013, **4** (4): 12–20.
[15] E. NEUMAN, *Sharp inequalities involving Neuman-Sándor and logarithmic means*, J. Math. Inequal., 2013, **7** (3): 413–419.
[16] E. NEUMAN AND J. SÁNDOR, *Bounds for the quotients of differences of certain bivariate means*, Adv. Stud. Contemp. Math., 2013, **23** (1): 61–67.
[17] Y.-M. CHU, M.-K. WANG AND Z.-K. WANG, *Best possible inequalities among harmonic, geometric, logarithmic and Seiffert means*, Math. Inequal. Appl., 2012, **15** (2): 415-433.
[18] W.-M. QIAN AND Y.-M. CHU, *Optimal bounds for Neuman means in terms of geometric, arithmetic and quadratic means*, J. Inequal. Appl., 2014, 2014: 175, 13 pages.

[19] Z.-H. Yang, Y.-Q. Song and Y.-M. Chu, *Monotonicity of the ratio of the power and second Seiffert means with applications*, Abstr. Appl. Anal., 2014, Article ID 840130, 4 pages.

[20] Y.-M. Chu, H. Wang and T.-H. Zhao, *Sharp bounds for Neuman mean in terms of the quadratic and second Seiffert means*, J. Inequal. Appl., 2014, 2014: 299, 14 pages.

[21] Y.-M. Chu and W.-M. Qian, *Refinements of bounds for Neuman means*, Abstr. Appl. Anal., 2014, Article ID 354132, 8 pages.

[22] E. Neuman, *On some means derived from the Schwab-Borchardt mean*, J. Math. Inequal., 2014, **8** (1): 171–183.

[23] E. Neuman, *On some means derived from the Schwab-Borchardt mean II*, J. Math. Inequal., 2014, **8** (2): 359–368.

[24] E. Neuman, *On a new bivariate mean*, Aequat. Math., 2014, **88** (3): 277–289.

[25] S.-B. Chen, Z.-Y. He, Y.-M. Chu, Y.-Q. Song and X.-J. Tao, *Note on certain inequalities for Neuman means*, J. Inequal. Appl., **2014**, 2014: 370, 10 pages.

[26] Z.-J. Guo, Y.-M. Chu, Y.-Q. Song and X.-J. Tao, *Sharp bounds for Neuman means by harmonic, arithmetic, and contraharmonic means*, Abstr. Appl. Anal., **2014**, Article ID 914242, 8 pages.

[27] Y. Zhang, Y.-M. Chu and Y.-L. Jiang, *Sharp geometric mean bounds for Neuman means*, Abstr. Appl. Anal., **2014**, Article ID 949815, 6 pages.

[28] G. D. Anderson, M. K. Vamanamurthy and M. Vuorinen, *Conformal Invariants, Inequalities, and Quasiconformal Maps*, John Wiley & Sons, New York, 1977.

[29] G. D. Anderson, S.-L. Qiu, M. K. Vamanamurthy and M. Vuorinen, *Generalized elliptic integrals and modular equations*, Pacific J. Math., 1997, **192** (1): 1–37.

[30] S. Simić and M. Vuorinen, *Landen inequalities for zero-balanced hypergeometric functions*, Abstr. Appl. Anal., **2012**, Article ID 932061, 11 pages.