

## ASYMPTOTIC FORMULAS FOR THE GAMMA FUNCTION BY GOSPER

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*Abstract.* The main aim of this paper is to give two general asymptotic expansions for the gamma function, which include the Gosper formula as their special cases. Furthermore, we present an inequality for the gamma function.

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