GENERALIZATION OF LEVINSON'S INEQUALITY

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Abstract. Mercer [5] gave a generalization of Levinson's inequality that replaces the assumption of symmetry of the two sequences with a weaker assumptions of equality of variances. Witkowski [10] further loosened this assumption and extended the result to the class of 3-convex functions.

We generalize these results to a newly defined, larger class of functions. We also prove the converse in case the function is continuous. In particular, we show that if Levinson's inequality holds under Mercer's assumptions, then the function is 3-convex.

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