

WEIGHTED SHARP FUNCTION ESTIMATE AND BOUNDEDNESS FOR COMMUTATOR ASSOCIATED WITH SINGULAR INTEGRAL OPERATOR SATISFYING A VARIANT OF HÖRMANDER'S CONDITION

XIAOSHA ZHOU

Abstract. In this paper, we establish a weighted sharp maximal function estimate for the commutator associated with the singular integral operator satisfying a variant of Hörmander's condition. As an application, we obtain the weighted boundedness of the commutators on Lebesgue and Morrey spaces.

Mathematics subject classification (2010): 42B20, 42B25.

Keywords and phrases: commutator; singular integral operator; sharp maximal function; Morrey space.

REFERENCES

- [1] S. BLOOM, *A commutator theorem and weighted BMO*, Trans. Amer. Math. Soc., **292** (1985), 103–122.
- [2] S. CHANILLO, *A note on commutators*, Indiana Univ. Math. J., **31** (1982), 7–16.
- [3] R. COIFMAN AND R. ROCHBERG, *Another characterization of BMO*, Proc. Amer. Math. Soc., **79**, 249–254 (1980).
- [4] R. R. COIFMAN, R. ROCHBERG AND G. WEISS, *Factorization theorems for Hardy spaces in several variables*, Ann. of Math., **103** (1976), 611–635.
- [5] G. DI FAZIO AND M. A. RAGUSA, *Commutators and Morrey spaces*, Boll. Un. Mat. Ital., **5-A** (7)(1991), 323–332.
- [6] G. DI FAZIO AND M. A. RAGUSA, *Interior estimates in Morrey spaces for strong solutions to nondivergence form equations with discontinuous coefficients*, J. Func. Anal., **112** (1993), 241–256.
- [7] J. GARCIA-CUERVA, *Weighted H^p spaces*, Dissert. Math., **162** (1979).
- [8] J. GARCIA-CUERVA AND J. L. RUBIO DE FRANCIA, *Weighted norm inequalities and related topics*, North-Holland Math., **16**, Amsterdam, 1985.
- [9] D. J. GRUBB AND C. N. MOORE, *A variant of Hörmander's condition for singular integrals*, Colloq. Math., **73**, 165–172 (1997).
- [10] B. HU AND J. GU, *Necessary and sufficient conditions for boundedness of some commutators with weighted Lipschitz spaces*, J. of Math. Anal. and Appl., **340** (2008), 598–605.
- [11] S. JANSON, *Mean oscillation and commutators of singular integral operators*, Ark. Math., **16** (1978), 263–270.
- [12] L. Z. LIU, *Interior estimates in Morrey spaces for solutions of elliptic equations and weighted boundedness for commutators of singular integral operators*, Acta Math. Scientia, **25(B)** (1), 89–94 (2005).
- [13] T. MIZUHARA, *Boundedness of some classical operators on generalized Morrey spaces*, in “Harmonic Analysis”, Proceedings of a conference held in Sendai, Japan, 1990, 183–189.
- [14] M. PALUSZYNSKI, *Characterization of the Besov spaces via the commutator operator of Coifman, Rochberg and Weiss*, Indiana Univ. Math. J., **44** (1995), 1–17.
- [15] J. PEETRE, *On convolution operators leaving $L^{p,\lambda}$ -spaces invariant*, Ann. Mat. Pura. Appl., **72** (1966), 295–304.
- [16] J. PEETRE, *On the theory of $L^{p,\lambda}$ -spaces*, J. Func. Anal., **4** (1969), 71–87.
- [17] C. PÉREZ, *Endpoint estimate for commutators of singular integral operators*, J. Func. Anal., **128** (1995), 163–185.

- [18] C. PÉREZ AND R. TRUJILLO-GONZALEZ, *Sharp weighted estimates for multilinear commutators*, J. London Math. Soc., **65** (2002), 672–692.
- [19] E. M. STEIN, *Harmonic analysis: real variable methods, orthogonality and oscillatory integrals*, Princeton Univ. Press, Princeton NJ, 1993.
- [20] R. TRUJILLO-GONZALEZ, *Weighted norm inequalities for singular integral operators satisfying a variant of Hörmander's condition*, Comment. Math. Univ. Carolin., **44**, 137–152 (2003).