

## THE GENERALIZED $L_p$ -WINTERNITZ PROBLEM

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**Abstract.** This article introduced the notion of the  $(i, j)$  type  $L_p$ -affine surface area of convex body in  $\mathbb{R}^n$ , and discussed its some proposition. In addition, we consider the more general  $L_p$ -Winternitz monotonicity problem about the  $(i, 0)$  type  $L_p$ -affine surface area and  $i$ th  $L_p$ -projection body in  $\mathbb{R}^n$ , and get a positive answer in all dimensions.

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