

## CAUCHY TYPE MEANS ON ONE-PARAMETER $C_0$ -GROUP OF OPERATORS

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*Abstract.* A new theory of power means is introduced on a  $C_0$ -group of continuous linear operators. A mean value theorem is proved, which builds the basis of the procedure to obtain Cauchy-type power means on a  $C_0$ -group of continuous linear operators.

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### REFERENCES

- [1] L. ALI KHAN, *Mean value theorem in topological vector spaces*, C. R. Math. Rep. Acad. Sci. Canada, **19**, 1 (1997), 24–27.
- [2] M. ANWAR, JOSIP PEČARIĆ, *Cauchy's means of Levinson type*, J. Inequal. Pure and Appl. Math., **9**, 4 (2008), 1–8.
- [3] M. ANWAR AND JOSIP PEČARIĆ, M. Rodić Lipanović, *Cauchy-type means for positive linear functionals*, Tamakang Journal Of Mathematics, **42**, 4 (2011), 511–530.
- [4] M. ANWAR AND JOSIP PEČARIĆ, *Cauchy's means for signed measures*, Bulletin of the Malaysian Mathematical Sciences Society, **34**, 1 (2011), 31–38.
- [5] M. ANWAR AND JOSIP PEČARIĆ, *Cauchy Means of Mercer's type*, Utilitas Mathematica, **84**, 2011, 201–208.
- [6] M. ANWAR AND JOSIP PEČARIĆ, *New Cauchy's Means*, J. Inequal. and Appl. Article ID 163202, 2008.
- [7] M. ANWAR, J. JAKŠETIĆ, J. PEČARIĆ AND ATIQ UR REHMAN, *Exponential convexity, positive semi-definite matrices and fundamental inequalities*, JMI, **4**, 2 (2010), 171–189.
- [8] A. BELLENI-MORANTE AND A. C. MCBRIDE, *Applied nonlinear semigroups: An introduction*, Wiley 1998.
- [9] G. D. BIKHOFF AND B. O. KOOPMAN, *Recent Contributions To The Ergodic Theory*, Proc Natl Acad Sci U S A. **18**, 3 (1932), 279–282.
- [10] IOAN I. VRABIE,  *$C_0$ -Semigroups and Applications*, North-Holland Mathematics studies **191**, Elsevier 2003.
- [11] K. J. ENGEL AND R. NAGEL, *One-Parameter Semigroups for Linear Evolution Equations*, Springer 2000.
- [12] E. LEACH AND M. SHOLANDER, *Multi-variable extended mean values*, J. Math. Anal. Appl., **104** (1984), 390–407.
- [13] R. NAGEL (Ed.), *One-parameter Semigroups of Positive Operators*, Lect. Notes in Math., **1184**,
- [14] J. P. ECKMANN AND D. RUELLE, *Ergodic theory of chaos and strange attractors*, Rev. Mod. Phys. **57** (1985), 617–656.
- [15] J. PEČARIĆ, M. RODIĆ LIPANOVIĆ AND H. M. SRIVASTAVA, *Some mean-value theorems of the Cauchy type*, Fractional Calculus And Applied Analysis, **9**, 2 (2006), 143–158, Springer-Verlag 1986.
- [16] K. YOSIDA, S. KAKUTANI, *Operator-Theoretical Treatment of Markoff's Process and Mean Ergodic Theorem*, Annals of Mathematics, Second Series, **42**, 1 (1941), 293–308.