

SOME ESTIMATES FOR HAUSDORFF OPERATORS

GUILIAN GAO, XIAOMEI WU, AMJAD HUSSAIN AND GUOPING ZHAO

Abstract. In this paper, we give some sufficient conditions for the boundedness of three types of Hausdorff operators on the Lebesgue spaces with power weights. In some cases, these conditions are also necessary and the corresponding operator norms are worked out. We extend and improve some known results in [6, 11].

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