

## TWO-WEIGHT INEQUALITIES FOR HARDY OPERATOR AND COMMUTATORS

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*Abstract.* For the maximal operator  $N$  related to the Hardy operator  $P$  and its adjoint  $Q$ , we give the characterizations for weights  $(u, v)$  such that  $N$  is bounded from  $L^p(v)$  to  $L^{p,\infty}(u)$  and from  $L^p(v)$  to  $L^p(u)$  respectively. We also obtain some  $A_p$  type conditions which are sufficient for the two-weight inequalities for the Hardy operator  $P$ , the adjoint operator  $Q$  and the commutators of these operators with CMO functions.

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