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A RESIDUAL–BASED POSTERIORI ERROR ESTIMATES FOR *hp* FINITE ELEMENT SOLUTIONS OF GENERAL BILINEAR OPTIMAL CONTROL PROBLEMS

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Abstract. In this paper, we investigate a residual-based posteriori error estimates for the *hp* finite element approximation of general optimal control problems governed by bilinear elliptic equations. By using the *hp* discontinuous Galerkin finite element approximation for the control and the *hp* finite element approximation for both the state and the co-state, we derive a posteriori upper error bounds for the optimal control problems governed by bilinear elliptic equations in L^2-H^1 norms. We also give a posteriori lower error bounds for the error estimate of the optimal control problems. These estimates can be readily used for constructing a reliable adaptive finite element approximation for such optimal control problems.

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