

ON AN EXTENSION OF SAKAGUCHI'S RESULT

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Abstract. We improve Pommerenke's result [15] by using a generalized lemma from [11]. It gives an extension of Sakaguchi's result too, [18]. Several applications of main theorems are presented. A part of them improves the previous results of this type. We consider also Sakaguchi's result under different type of the assumptions.

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