INEQUALITIES FOR THE GENERALIZED TRIGONOMETRIC, HYPERBOLIC AND JACOBIAN ELLIPTIC FUNCTIONS

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Abstract. This paper deals with the inequalities for the generalized trigonometric, hyperbolic and the Jacobian elliptic functions. These families of higher transcendental functions are of great importance in the studies of some problems that arose in the theory of differential equations. Among the main results established in this paper the Wilker- and Huygens- type inequalities for the functions under discussion are obtained.

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