OSTROWSKI AND TRAPEZOID TYPE INEQUALITIES
RELATED TO POMPEIU'S MEAN VALUE THEOREM

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Abstract. In this paper, some new Ostrowski and trapezoid type inequalities, which are related to Pompeiu's mean value theorem, are obtained for absolutely continuous functions. Some applications to special means and inequalities for f-divergence measures are also given.

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